Name of the Staff: Mr.B.Kandavel
Name of the Subject: Control Systems
Subject Code: BEEF185T30
Semester: V
Year: Third Year EEE (Regular)

COURSE PLAN

Topic No	Topic Name	No. of Periods	Proposed Date of Completion
UNIT – I : SYSTEM AND THEIR REPRESENTATION			
1	Basic elements of control systems	1	7/13/2021
2	open and close loop systems	1	7/13/2021
3	Differential equation	1	7/15/2021
4	Transfer function – Modelling of Electrical systems	1	7/15/2021
5	translational and rotational mechanical systems	1	7/20/2021
6	Block diagram reduction techniques	1	7/22/2021
7	Block diagram reduction techniques	1	7/22/2021
8	Signal flow graphs	1	7/27/2021
9	Signal flow graphs	1	7/27/2021
UNIT – II : TIME RESPONSE			
1	Time response	1	7/29/2021
2	types of input – I	1	7/29/2021
3	II order system response	1	8/3/2021
4	Time domain specifications	1	8/3/2021
5	Steady state error	1	8/5/2021
6	Error Constant	1	8/10/2021
7	Error coefficients	1	8/10/2021
8	Generalized error series	1	8/12/2021
9	Effect of P,PI,PD and PID modes of feedback control	1	8/12/2021
UNIT – III : FREQUENCY RESPONSE			
1	Frequency response	1	8/24/2021
2	Bode plot	1	8/24/2021
3	Bode plot	1	8/26/2021
4	Polar plot	1	8/26/2021
5	Polar plot	1	8/31/2021
6	Frequency domain specifications from plots	1	8/31/2021
7	Nichol’s chart	1	9/2/2021

8	Constant M and N circles	**1**	9/7/2021
9	Nyquist plot	**1**	9/7/2021
UNIT – IV : STABILITY AND COMPENSATOR DESIGN			
1	Characteristic equation	**1**	9/9/2021
2	BIBO stability	**1**	9/14/2021
3	Routh Hurwitz criterion	**1**	9/16/2021
4	Root locus technique Construction of Root locus	**1**	9/21/2021
5	Nyquist stability criterion	**1**	9/28/2021
6	Effect of Lag, Lead and lag-lead compensation on frequency response, Analysis using MATLAB	**1**	9/30/2021
UNIT – V : STATE VARIABLE ANALYSIS			
1	Concept of state variables	**2**	10/5/2021
2	State models for linear and time invariant systems	**2**	10/7/2021
3	solution of state and output equation in controllable canonical form	**2**	10/12/2021
4	concept of controllability and observability	**2**	10/19/2021
5	Effect of state feedback	**2**	10/21/2021

Year and Semester: III Year and Fifth Sem

Course Name: Control Systems

Course Code : BEEF185T30

Course Teacher: Mr.B.Kandavel

Topic No: Unit 1

Title: Differential equation - Transfer function – Modeling of Electrical systems, translational and rotational mechanical systems – Block diagram reduction techniques – Signal flow graphs

Aim and Objective: To study the Basics of systems, modeling of various kind of systems, detection of transfer function from the pictorial representation

Pre Test MCQ

1.Transfer function of a system is defined as the ratio of output to input in

a) Z-transformer
b) Fourier transform
c) **Laplace transform**
d) All of these

2.Traffic light system is the example of:

(a) Open-loop system

(b) Closed-loop system

(c) Both (a) and (b)

(d) None of these

3.The Static system can be defined as:

(a) Output of a system depends on the present as well as past input.

(b) Output of a system depends only on the received inputs.

(c) Output of the system depends on future inputs.

(d) Output of the system depends only on the present input.

4.In an open loop control system

(a) **Output is independent of control input**

(b) Output is dependent on control input

(c) Only system parameters have effect on the control output

(d) None of the above

5. A control system in which the control action is somehow dependent on the output is known as

(a) **Closed loop system**

(b) Semi closed loop system

(c) Open system

(d) None of the above

6. Which of the following is an open loop control system ?

(a) Field controlled D.C. motor

(b) Ward leonard control

(c) Metadyne

(d) Stroboscope

7. A car is moving at a constant speed of 50 km/h, which of the following is the feedback element for the driver ?

(a) Clutch

(b) Eyes

(c) Needle of the speedometer

(d) Steering wheel

(e) None of the above

8. An automatic toaster is a ______ loop control system.

(a) open

(b) closed

(c) partially closed

(d) any of the above

9. ___ is a closed loop system.

(a) **Auto-pilot for an aircraft**

(6) Direct current generator

(c) Car starter

(d) Electric switch

Prerequisites:

1) Knowledge about the Circuit Analysis
2) Knowledge about Laplace and Inverse Laplace Transforms
3) Knowledge about the Control Systems and Open loop and Closed loop Systems

Theory Behind:

Differential equation modeling of mechanical systems. There are two types of mechanical systems based on the type of motion.

- Translational mechanical systems
- Rotational mechanical systems

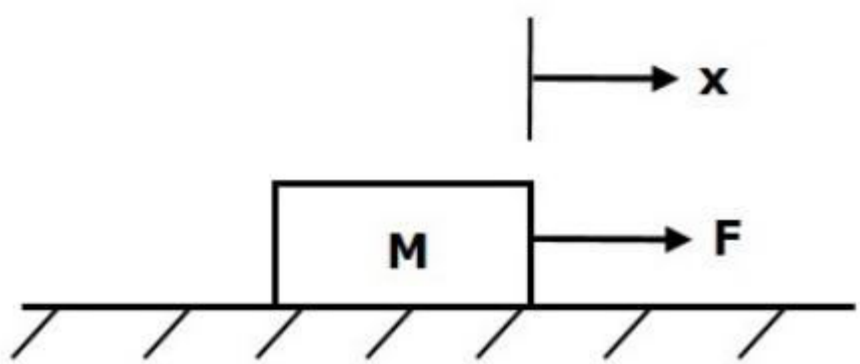
Modeling of Translational Mechanical Systems

Translational mechanical systems move along a straight line. These systems mainly consist of three basic elements. Those are mass, spring and dashpot or damper.

If a force is applied to a translational mechanical system, then it is opposed by opposing forces due to mass, elasticity and friction of the system. Since the applied force and the opposing forces are in opposite directions, the algebraic sum of the forces acting on the system is zero. Let us now see the force opposed by these three elements individually.

Ideal Mass Element

Mass is the property of a body, which stores kinetic energy. If a force is applied on a body having mass M, then it is opposed by an opposing force due to mass. This opposing force is proportional to the acceleration of the body. Assume elasticity and friction are negligible.



$F_M \propto a$

$$F_M = Ma = M\frac{d^2X}{dt^2}$$

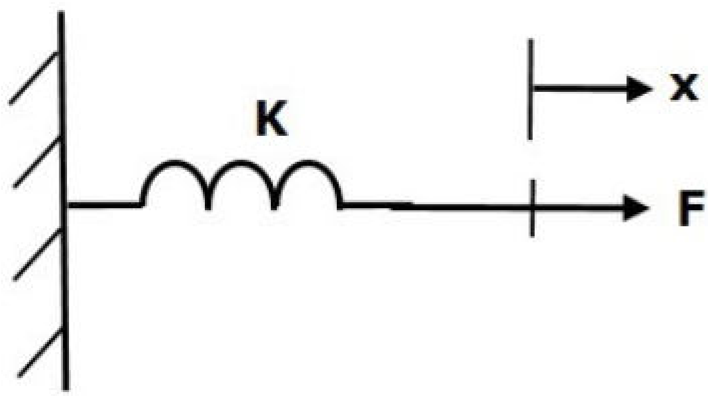
$$F = F_M = M\frac{d^2X}{dt^2}$$

Where,

- **F** is the applied force
- $\mathbf{F_m}$ is the opposing force due to mass
- **M** is mass
- **a** is acceleration
- **x** is displacement

Ideal Spring Element

Spring is an element, which stores **potential energy**. If a force is applied on spring **K**, then it is opposed by an opposing force due to elasticity of spring. This opposing force is proportional to the displacement of the spring. Assume mass and friction are negligible.



$F_K \propto x$

$F_K = Kx$

$F = F_K = Kx$

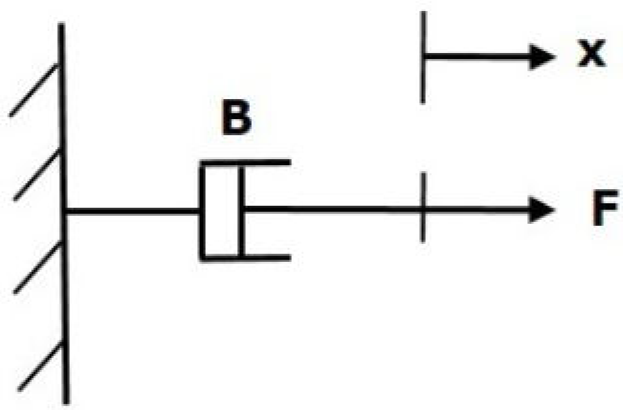
Where,

- **F** is the applied force
- $\mathbf{F_k}$ is the opposing force due to elasticity of spring
- **K** is spring constant

- **x** is displacement

Ideal Dashpot Element

If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_B \propto v$$

$$F_B = Bv = B\frac{dx}{dt}$$

$$F = F_B = B\frac{dx}{dt}$$

Where,

- $\mathbf{F_b}$ is the opposing force due to friction of dashpot
- **B** is the frictional coefficient
- **v** is velocity
- **x** is displacement

Modeling of Rotational Mechanical Systems

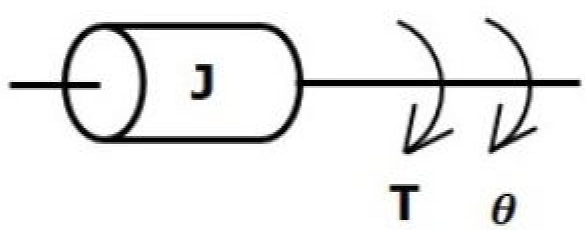
Rotational mechanical systems move about a fixed axis. These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.

If a torque is applied to a rotational mechanical system, then it is opposed by opposing torques due to moment of inertia, elasticity and friction of the system. Since the applied torque and the opposing torques are in opposite directions, the algebraic sum of torques acting on the system is zero. Let us now see the torque opposed by these three elements individually.

Moment of Inertia

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.

If a torque is applied on a body having moment of inertia **J**, then it is opposed by an opposing torque due to the moment of inertia. This opposing torque is proportional to angular acceleration of the body. Assume elasticity and friction are negligible.



$$T_J \propto \alpha$$

$$T_J = J\alpha = J\frac{d^2\theta}{dt^2}$$

$$T = T_J = J\frac{d^2\theta}{dt^2}$$

Where,

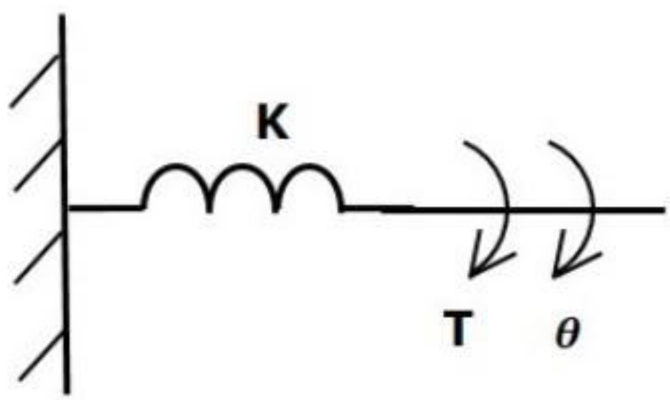
- **T** is the applied torque
- $\mathbf{T_j}$ is the opposing torque due to moment of inertia
- **J** is moment of inertia

- **α** is angular acceleration
- **θ** is angular displacement

Torsional Spring

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.

If a torque is applied on torsional spring **K**, then it is opposed by an opposing torque due to the elasticity of torsional spring. This opposing torque is proportional to the angular displacement of the torsional spring. Assume that the moment of inertia and friction are negligible.



$T_K \propto \theta$

$T_K = K\theta$

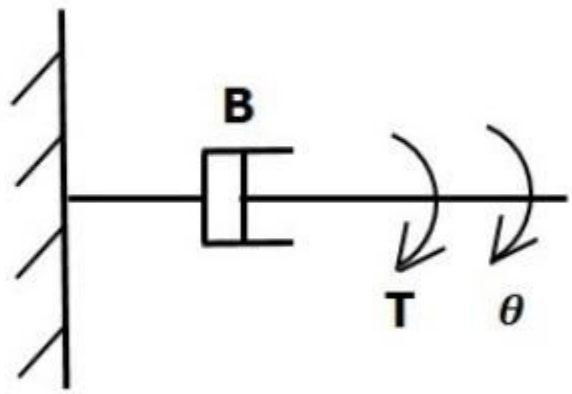
$T = T_K = K\theta$

Where,

- **T** is the applied torque
- **T_k** is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot. This opposing torque is proportional to the angular velocity of the body. Assume the moment of inertia and elasticity are negligible.



$T_B \propto \omega$

$$T_B = B\omega = B\frac{d\theta}{dt}$$

$$T = T_B = B\frac{d\theta}{dt}$$

Where,

- $\mathbf{T_b}$ is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement

Analogous systems

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

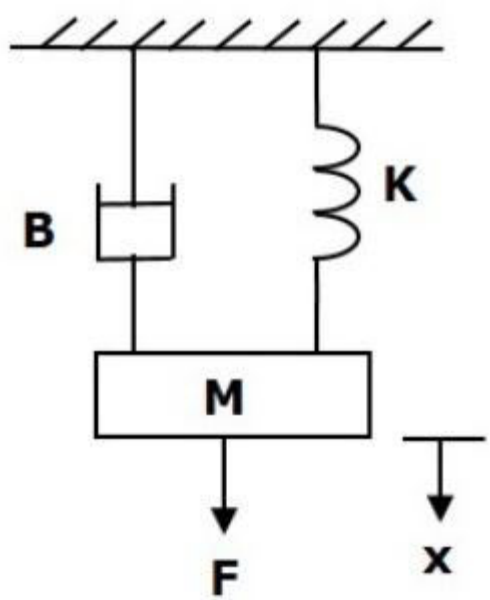
- The two systems are physically different
- Differential equation modeling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.

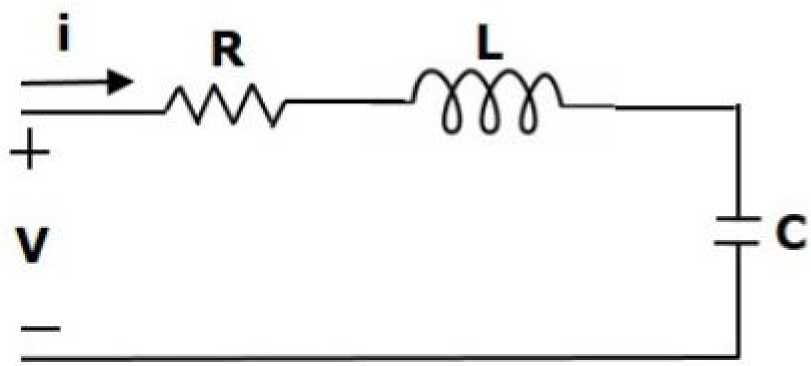


The **force balanced equation** for this system is

$$F = F_M + F_B + F_K$$

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is V volts and the current flowing through the circuit is i Amps.



Mesh equation for this circuit is

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

substitute $i = \frac{dq}{dt}$ in the above equation

$$V = R\frac{dq}{dt} + L\frac{d}{dt}\left(\frac{dq}{dt}\right) + \frac{1}{C}\int \frac{dq}{dt}\,dt$$

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

$$V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C}$$

Comparing Mechanical System equation with Electrical System. we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

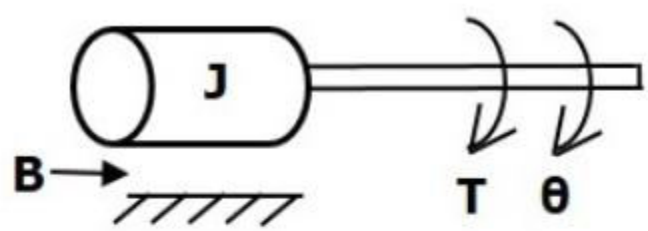
Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance (1/C)
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Torque Voltage Analogy

In this analogy, the mathematical equations of **rotational mechanical system** are compared with mesh equations of the electrical system.

Rotational mechanical system is shown in the following figure.



The torque balanced equation is

$$T = T_J + T_B + T_K$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

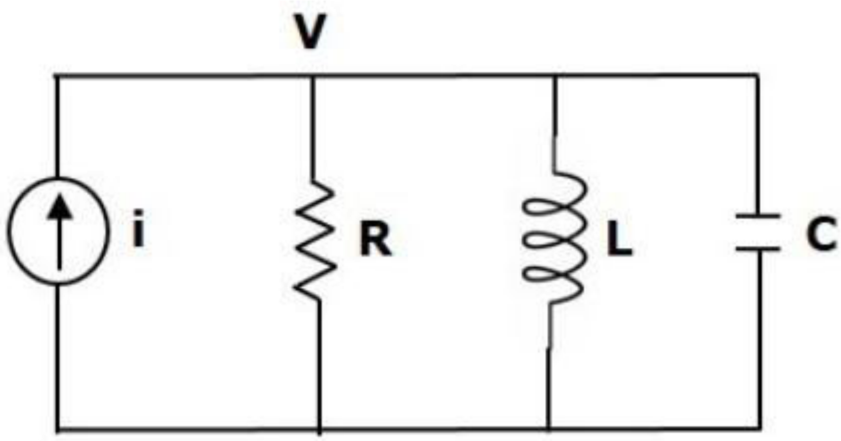
By Comparing Voltage equation of electrical circuit with Torque equation we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1/C)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$i = \frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

Substitute $v = \frac{d\phi}{dt}$ in the above eqn

$$i = \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} + C\frac{d^2\phi}{dt^2}$$

$$i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}$$

Comparing current eqn with force eqn obtained from translational systems. we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance(1/R)
Spring constant(K)	Reciprocal of Inductance(1/L)(1L)

Displacement(x)	Magnetic Flux(ϕ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

Torque Current Analogy

In this analogy, the mathematical equations of the **rotational mechanical system** are compared with the nodal mesh equations of the electrical system.

Comparing current eqn with torque eqn obtained from **rotational** systems with current eqn, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance(1/R)
Torsional spring constant(K)	Reciprocal of Inductance(1/L)
Angular displacement(θ)	Magnetic flux(ϕ)
Angular velocity(ω)	Voltage(V)

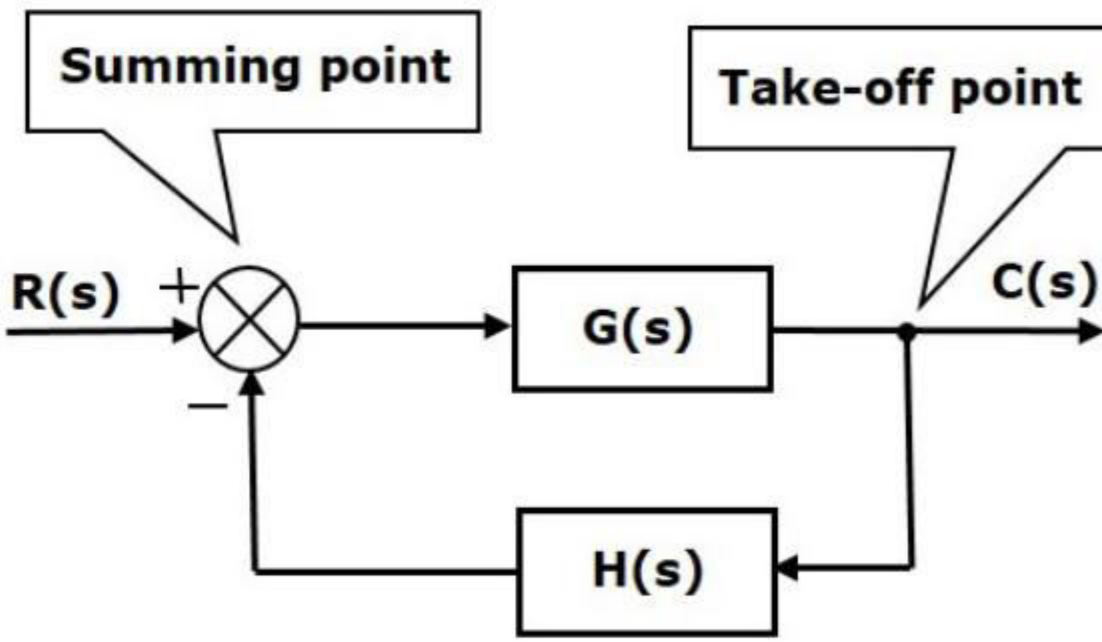
Block Diagram in control systems

Any system can be described by a set of differential equations, or it can be represented by the schematic diagram that contains all the components and their connections. However, these methods do not work for complicated systems.

The Block diagram representation is a combination of these two methods. A block diagram is a representation of a system using blocks.

Basic Elements of Block Diagram

The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements



The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input R(s), output C(s) and the transfer function G(s).

For representing any system using block diagram, it is necessary to find the transfer function of the system which is the ratio of Laplace of output to Laplace of input.

Where

R(s) = Input

C(s) = Output

G(s) = Transfer function

Then, the system can be represented as

$$G(s) = \frac{C(s)}{R(s)}$$

$$C(s) = G(s).R(s)$$

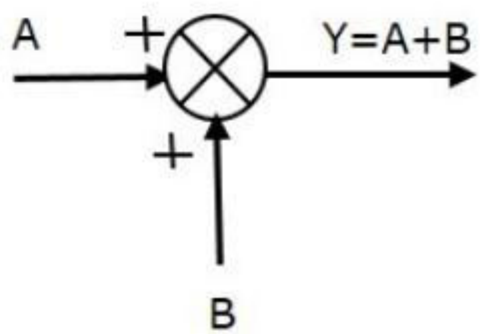
Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

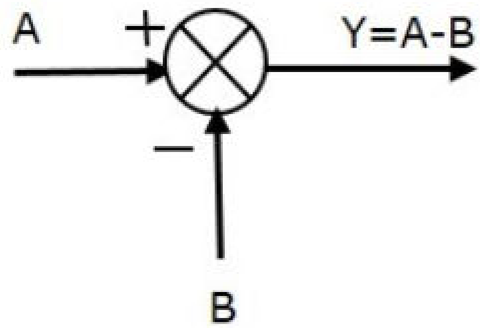
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

i.e., Y = A + B.



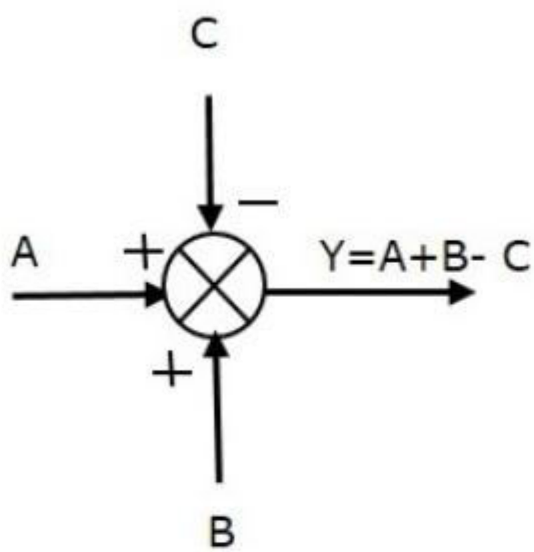
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negative sign. So, the summing point produces the output **Y** as the **difference of A and B**.

Y = A + (-B) = A - B.



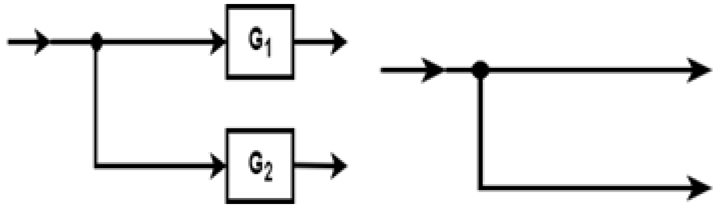
The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

$Y = A + B + (-C) = A + B - C.$



Take off Point:

When there is more than one block, and we want to apply the same input to all the blocks, we use a take-off point. By the use of a take-off point, the same input propagates to all the blocks without affecting its value. Representation of same input to more than one block is shown in the below diagram.



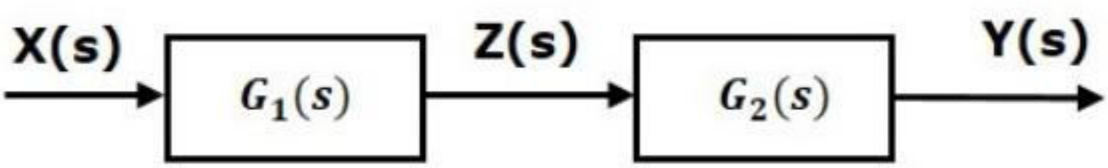
Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

Basic Connections for Blocks

There are three basic types of connections between two blocks.

Series Connection

Series connection is also called **cascade connection**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in series.



For this combination, we will get the output Y(s) as

$Y(s) = G_2(s)Z(s)$

Where $Z(s) = X(s)G_1(s)$

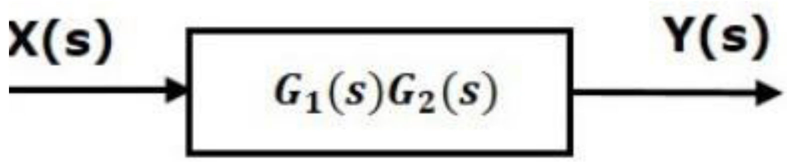
$Y(s) = G_2(s)[X(s)G_1(s)]$

$Y(s) = G_1(s)G_2(s)X(s)$

$Y(s) = \{G_1(s)G_2(s)\}X(s)$

Compare this equation with the standard form of the output equation, Y(s)=G(s)X(s). Where, G(s)=G1(s)G2(s)

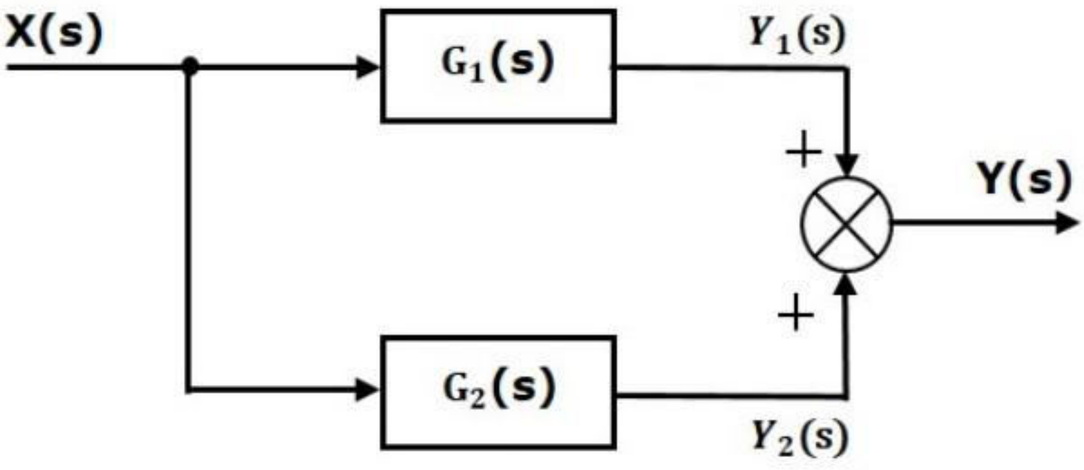
That means we can represent the **series connection** of two blocks with a single block. The transfer function of this single block is the **product of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent series connection of 'n' blocks with a single block. The transfer function of this single block is the product of the transfer functions of all those 'n' blocks.

Parallel Connection

The blocks which are connected in **parallel** will have the **same input**. In the following figure, two blocks having transfer functions G1(s) and G2(s) are connected in parallel. The outputs of these two blocks are connected to the summing point.



For this combination, we will get the output Y(s)Y(s) as

$Y(s) = Y_1(s) + Y_2(s)$

Where $Y_1(s) = G_1(s)X(s)$ and $Y_2(s) = G_2(s)X(s)$

$Y(s) = G_1(s)X(s) + G_2(s)X(s)$

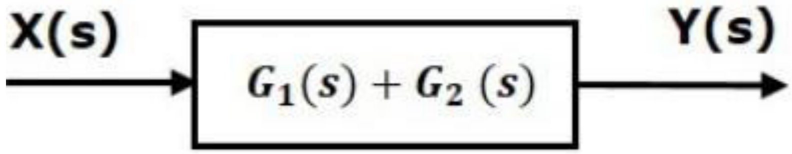
$Y(s) = \{G_1(s) + G_2(s)\}X(s)$

Compare this equation with the standard form of the output equation,

$C(s) = G(s).R(s)$

where $G(s) = G_1(s) + G_2(s)$

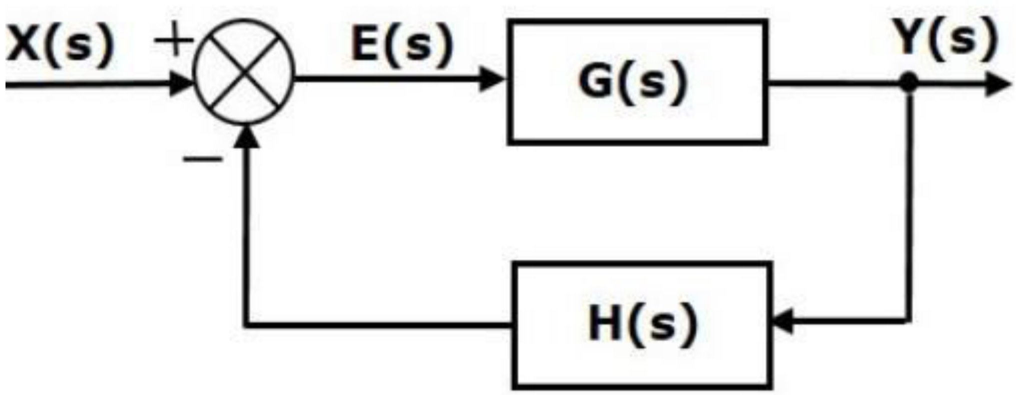
That means we can represent the **parallel connection** of two blocks with a single block. The transfer function of this single block is the **sum of the transfer functions** of those two blocks. The equivalent block diagram is shown below.



Similarly, you can represent parallel connection of ‘n’ blocks with a single block. The transfer function of this single block is the algebraic sum of the transfer functions of all those ‘n’ blocks.

Feedback Connection

As we discussed in previous chapters, there are two types of **feedback** — positive feedback and negative feedback. The following figure shows negative feedback control system. Here, two blocks having transfer functions G(s) and H(s) form a closed loop.



The output of the summing point is -

$$E(s) = X(s) - Y(s)H(s)$$

The output Y(s) is

$$Y(s) = E(s)G(s)$$

Substitute E(s) value in the above equation

$$Y(s) = \{X(s) - Y(s)H(s)\}G(s)$$

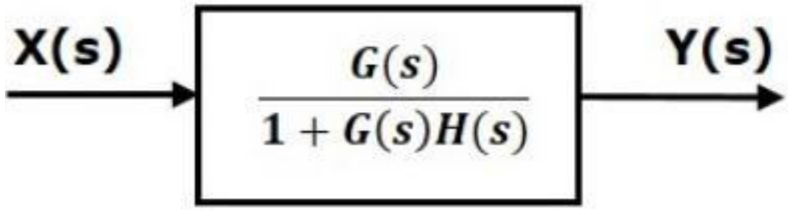
$$Y(s) + Y(s)H(s)G(s) = X(s)G(s)$$

$$Y(s)(1 + H(s)G(s)) = X(s)G(s)$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Therefore, the negative feedback closed loop transfer function is $\frac{G(s)}{1+G(s)H(s)}$

This means we can represent the negative feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the negative feedback. The equivalent block diagram is shown below.



Similarly, you can represent the positive feedback connection of two blocks with a single block. The transfer function of this single block is the closed loop transfer function of the positive feedback, i.e., $\frac{G(s)}{1-G(s)H(s)}$

Block Diagram Algebra for Summing Points

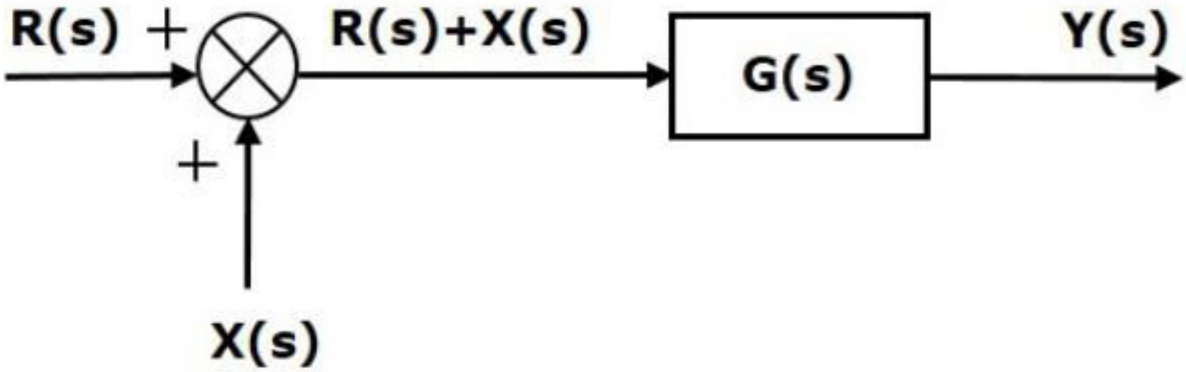
There are two possibilities of shifting summing points with respect to blocks −

- Shifting summing point after the block
- Shifting summing point before the block

Let us now see what kind of arrangements need to be done in the above two cases one by one.

Shifting Summing Point After the Block

Consider the block diagram shown in the following figure. Here, the summing point is present before the block.



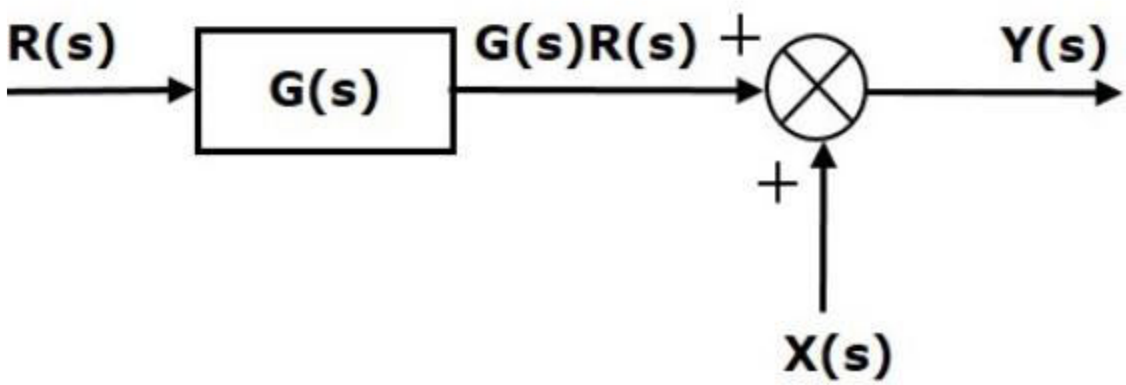
Summing point has two inputs R(s) and X(s). The output of it is {R(s)+X(s)}

So, the input to the block G(s) is {R(s)+X(s) and the output of it is

$Y(s) = G(s)\{R(s) + X(s)\}$

$Y(s) = G(s)R(s) + G(s)X(s)$

Now, shift the summing point after the block. This block diagram is shown in the following figure.

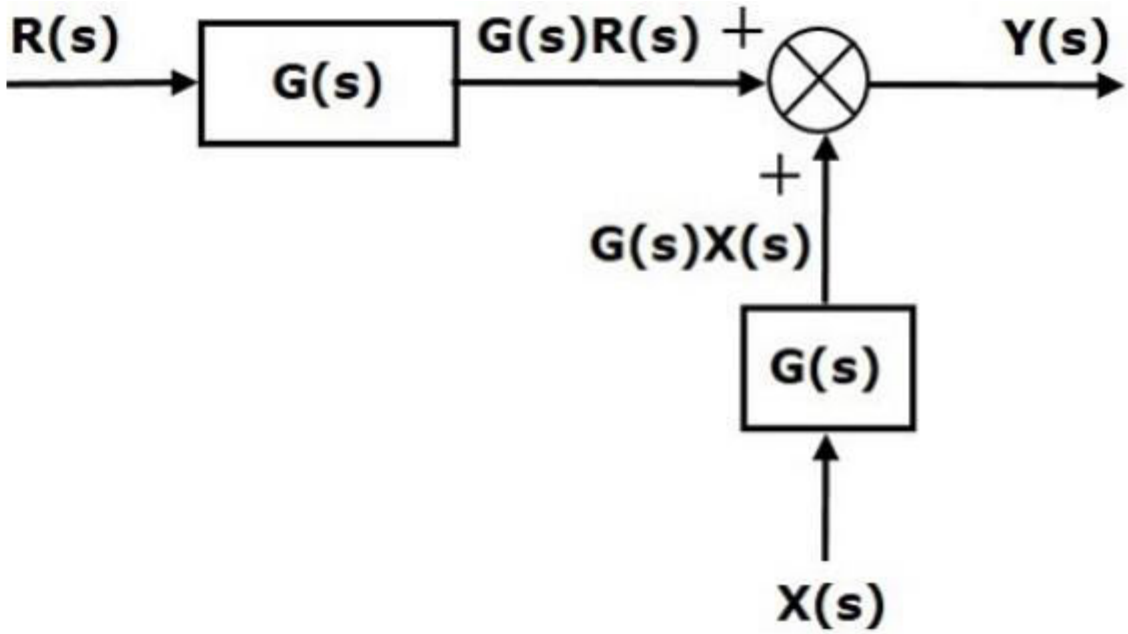


Output of the block G(s) is G(s)R(s)

The output of the summing point is

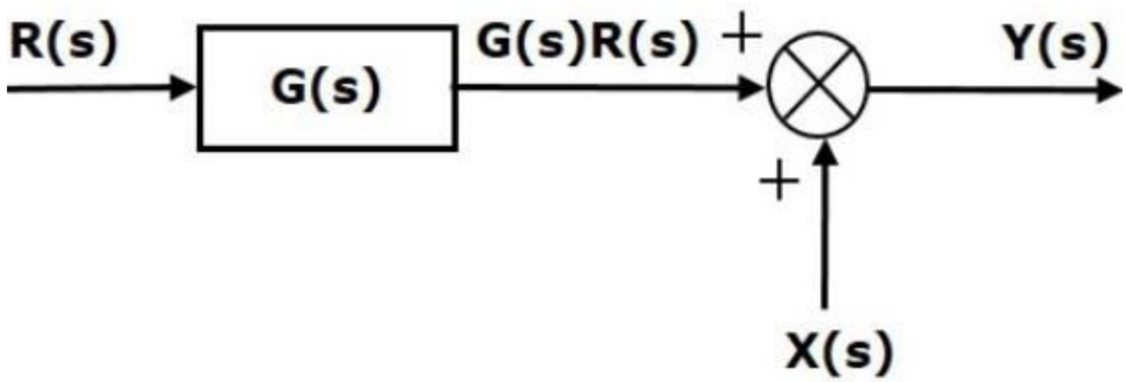
$Y(s) = G(s)R(s) + X(s)$

The first term G(s)R(s)is same in both the equations. But, there is difference in the second term. In order to get the second term also same, we require one more block G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s).This block diagram is shown in the following figure.



Shifting Summing Point Before the Block

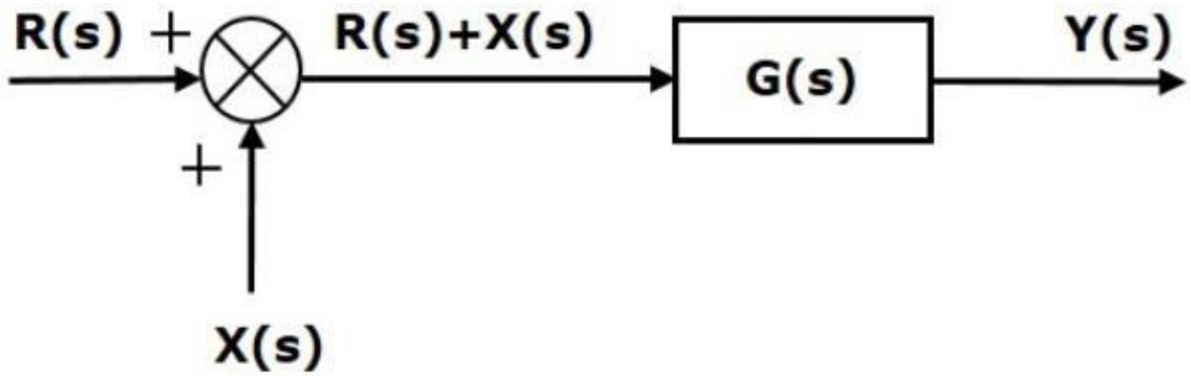
Consider the block diagram shown in the following figure. Here, the summing point is present after the block



Output of this block diagram is -

$Ys) = G(s)R(s) + X(s)$

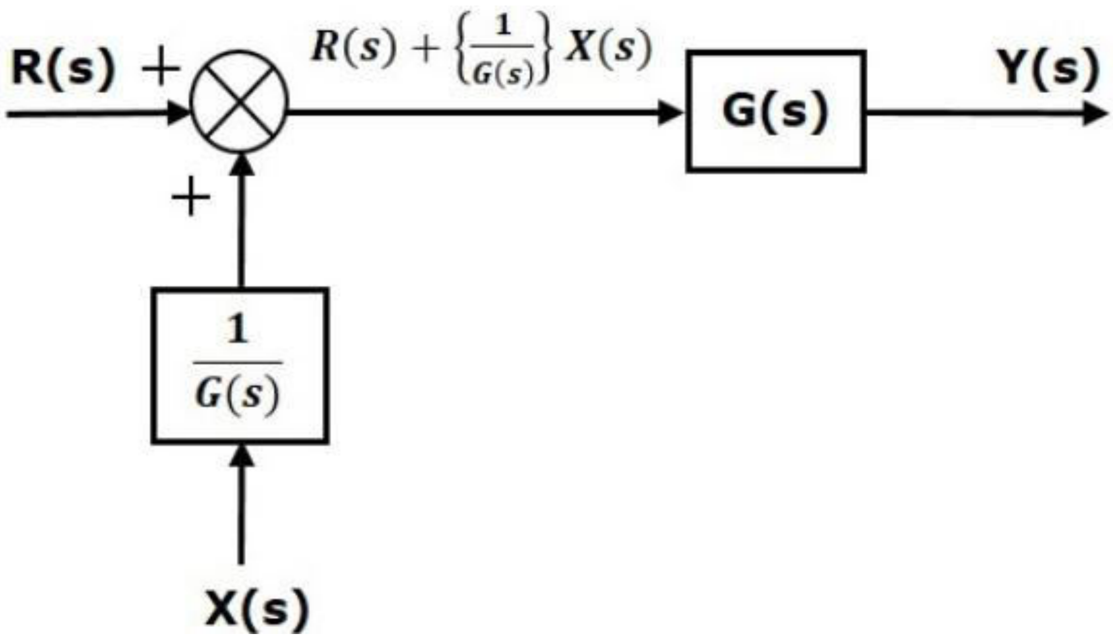
Now, shift the summing point before the block. This block diagram is shown in the following figure.



Output of this block diagram is

$Y(s) = G(s)\,[R(s) + X(s)]$

The first term G(s)R(s) is same in both equations. But, there is difference in the second term. In order to get the second term also same, we require one more block 1/G(s). It is having the input X(s) and the output of this block is given as input to summing point instead of X(s). This block diagram is shown in the following figure.



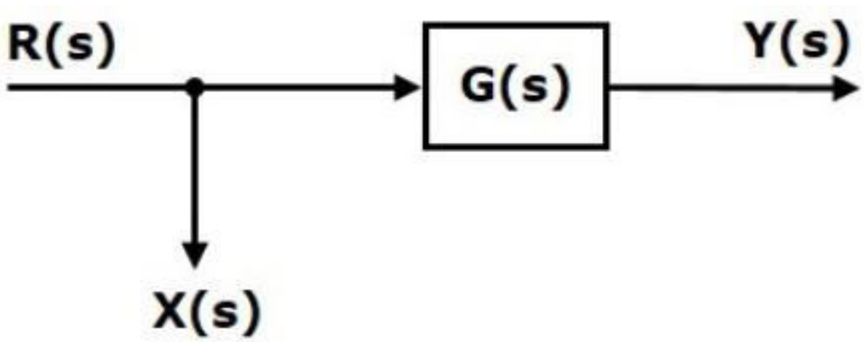
Block Diagram Algebra for Take-off Points

There are two possibilities of shifting the take-off points with respect to blocks −

- Shifting take-off point after the block
- Shifting take-off point before the block

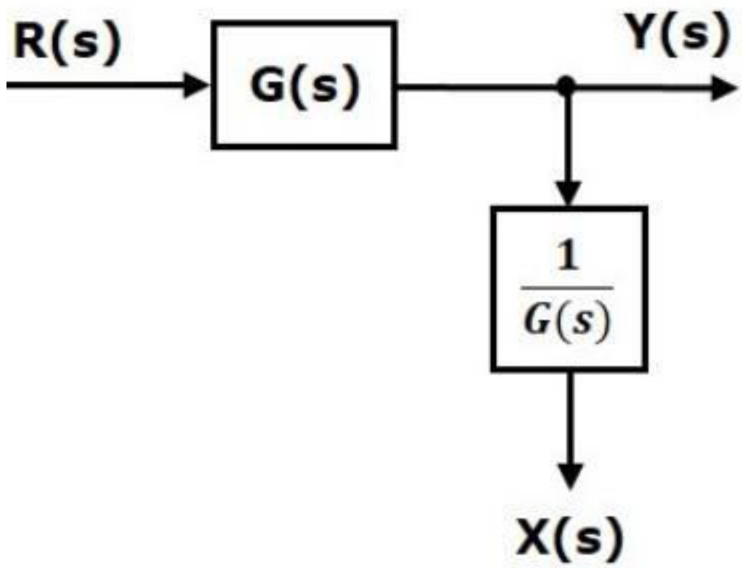
Shifting Take-off Point After the Block

Consider the block diagram shown in the following figure. In this case, the take-off point is present before the block.



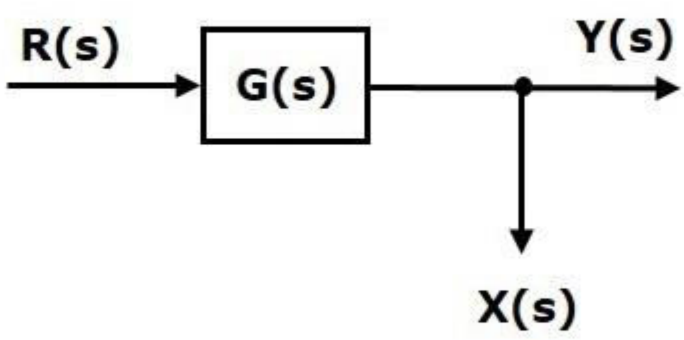
Here, X(s)=R(s) and Y(s)=G(s)R(s)

When you shift the take-off point after the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get the same X(s) value, we require one more block $\frac{1}{G(s)}$. It is having the input Y(s)Y(s) and the output is X(s) This block diagram is shown in the following figure.



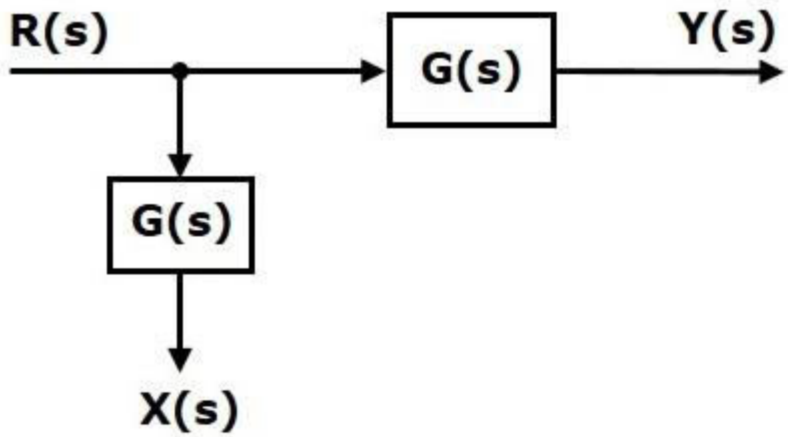
Shifting Take-off Point Before the Block

Consider the block diagram shown in the following figure. Here, the take-off point is present after the block.



Here, X(s)=Y(s)=G(s)R(s)

When you shift the take-off point before the block, the output Y(s) will be same. But, there is difference in X(s) value. So, in order to get same X(s) value, we require one more block G(s). It is having the input R(s) and the output is X(s). This block diagram is shown in the following figure.



Block Diagram Reduction Rules

Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1** − Check for the blocks connected in series and simplify.
- **Rule 2** − Check for the blocks connected in parallel and simplify.
- **Rule 3** − Check for the blocks connected in feedback loop and simplify.
- **Rule 4** − If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5** − If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6** − Repeat the above steps till you get the simplified form, i.e., single block.

The transfer function present in this single block is the transfer function of the overall block diagram.

Signal Flow Graph

Signal flow graph is a graphical representation of algebraic equations. In this chapter, let us discuss the basic concepts related signal flow graph and also learn how to draw signal flow graphs.

Basic Elements of Signal Flow Graph

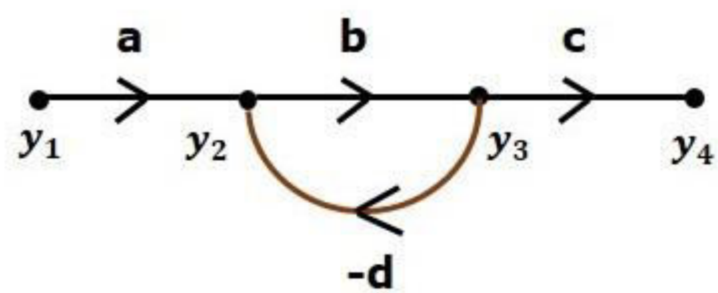
Nodes and branches are the basic elements of signal flow graph.

Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node** − It is a node, which has only outgoing branches.
- **Output Node** − It is a node, which has only incoming branches.
- **Mixed Node** − It is a node, which has both incoming and outgoing branches.

Let us consider the following signal flow graph to identify these nodes.



- The **nodes** present in this signal flow graph are $\mathbf{y_1, y_2, y_3}$ and $\mathbf{y_4}$.
- $\mathbf{y_1}$ and $\mathbf{y_4}$ are the **input node** and **output node** respectively.
- $\mathbf{y_2}$ and $\mathbf{y_3}$ are **mixed nodes**.

Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

$$y_4 = a_{34}y_3$$

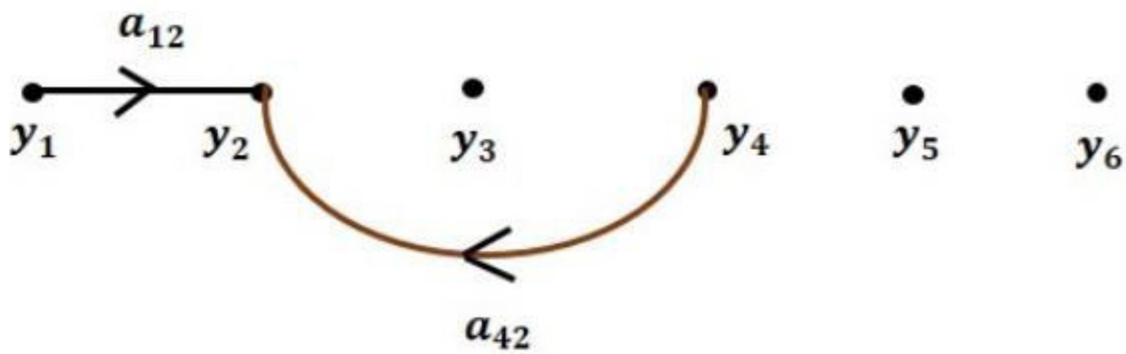
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

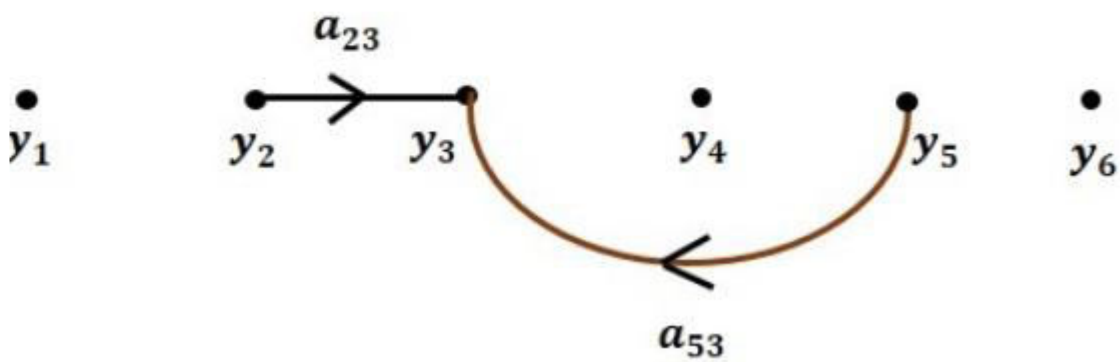
There will be six **nodes** (y_1, y_2, y_3, y_4, y_5 and y_6) and eight **branches** in this signal flow graph. The gains of the branches are a_{12}, a_{23}, a_{34}, a_{45}, a_{56}, a_{42}, a_{53} and a_{35}.

To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below.

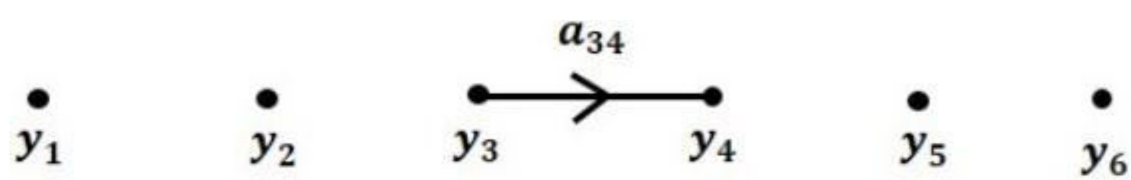
Step 1 − Signal flow graph for $y_2 = a_{12}y_1 + a_{42}y_4$ is shown in the following figure.



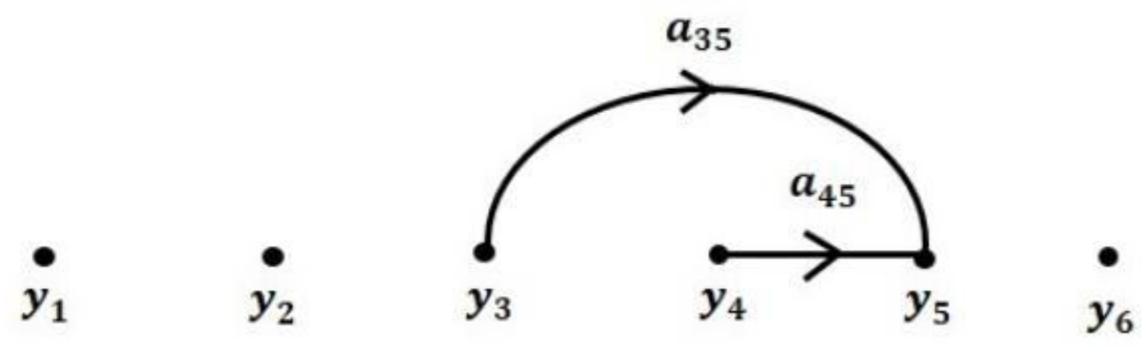
Step 2 − Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.



Step 3 − Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.

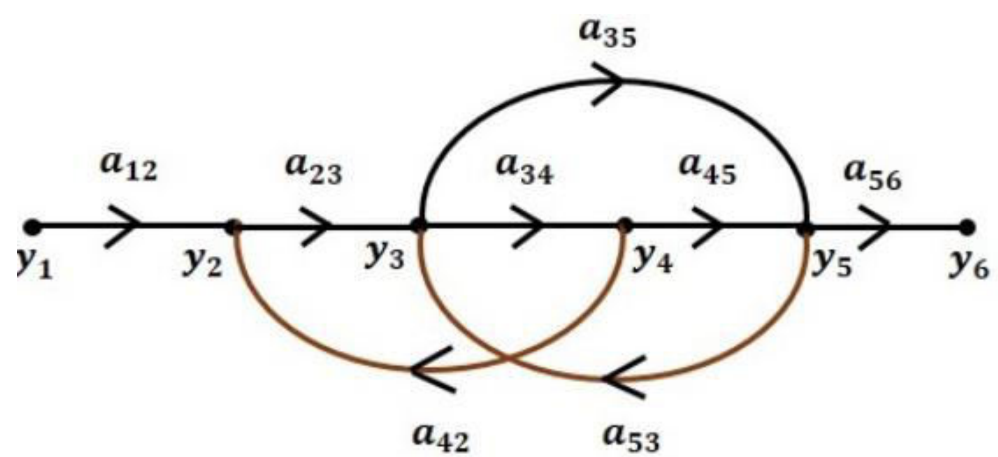


Step 4 − Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Step 5 − Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure

Step 6 − Signal flow graph of overall system is shown in the following figure



Conversion of Block Diagrams into Signal Flow Graphs

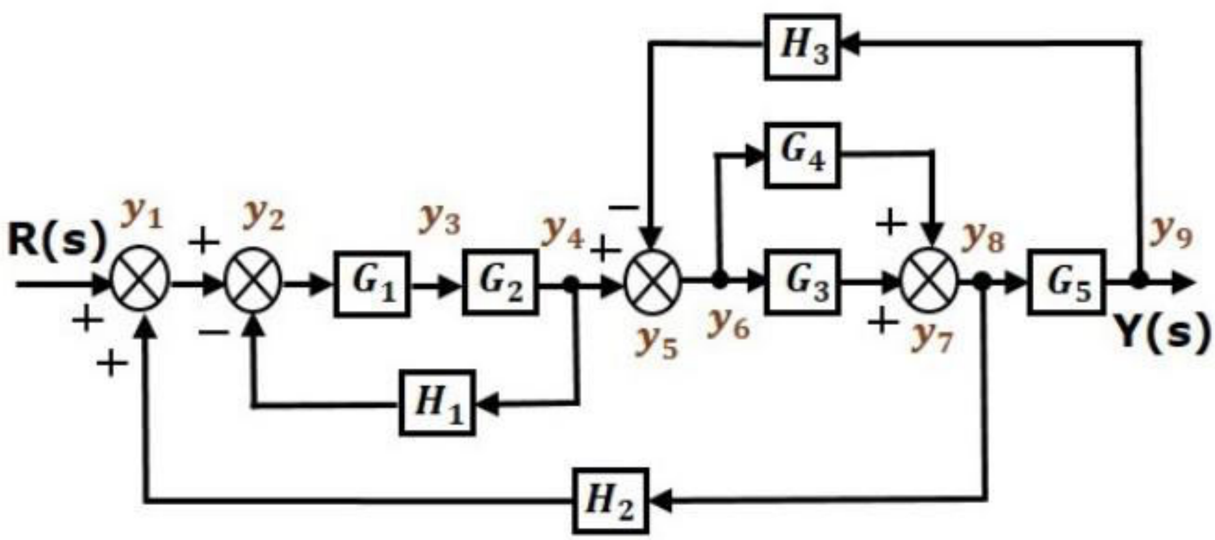
Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.

- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one. **For example**, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

Example

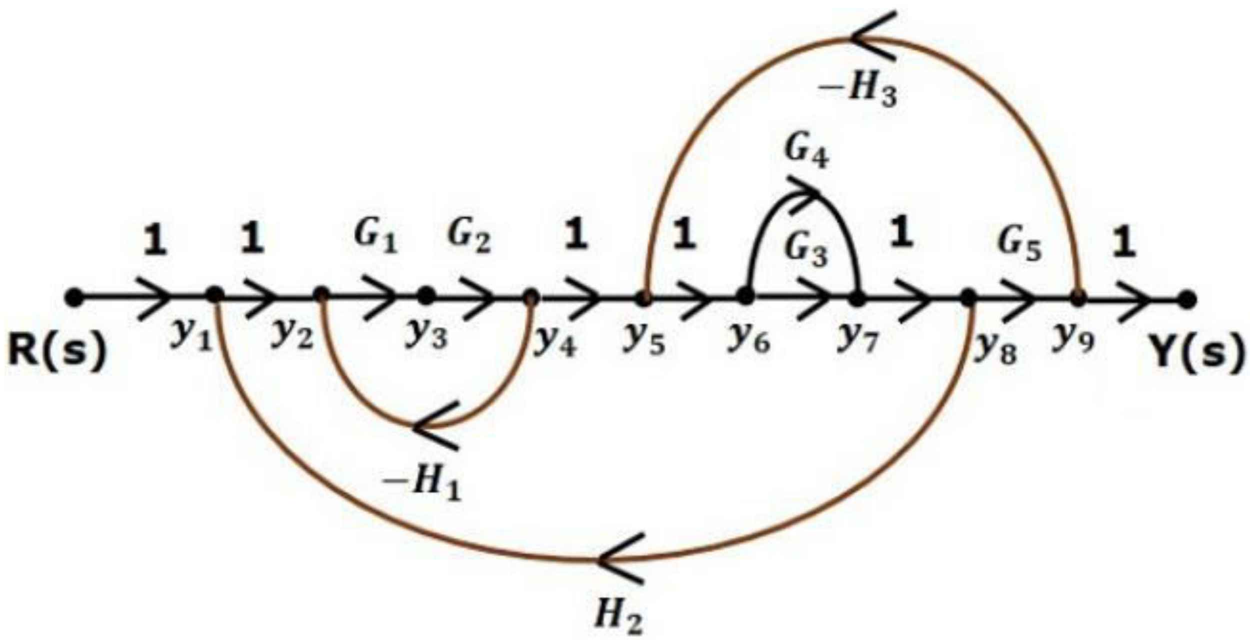
Let us convert the following block diagram into its equivalent signal flow graph.



Represent the input signal R(s) and output signal C(s) of block diagram as input node R(s) and output node C(s) of signal flow graph.

Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1 and G2.

The following figure shows the equivalent signal flow graph.



Suppose there are 'N' forward paths in a signal flow graph. The gain between the input and the output nodes of a signal flow graph is nothing but the **transfer function** of the system. It can be calculated by using Mason's gain formula.

Mason's gain formula is

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K$$

Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between *R*(*s*) and *C*(*s*)
- $\mathbf{P_K}$ is the K^{th} forward path gain

$$\Delta = 1 - (\text{Sum of Individual loop gains})$$
$$+ (\text{Sum of gain products of all possible two non touching loops})$$
$$- (\text{Sum of gain products of all possible three non touching loops})$$
$$+ \cdots$$

Δ_K is obtained from Δ by removing the loops which are touching the K^{th} forward path

Post Test MCQ

1. When deriving the transfer function of a linear element

a) Both initial conditions and loading are taken into account

b) Initial conditions are taken into account but the element is assumed to be not loaded

c) **Initial conditions are assumed to be zero but loading is taken into account**

d) Initial conditions are assumed to be zero and the element is assumed to be not loaded

2. The overall transfer function from block diagram reduction for cascaded blocks is :

a) Sum of individual gain

b) **Product of individual gain**

c) Difference of individual gain

d) Division of individual gain

3. The overall transfer function of two blocks in parallel are :

a) **Sum of individual gain**

b) Product of individual gain

c) Difference of individual gain

d) Division of individual gain

4. Transfer function of the system is defined as the ratio of Laplace output to Laplace input considering initial conditions________

a) 1

b) 2

c) **0**

d) infinite

5. A node having only outgoing branches.

a) **Input node**

b) Output node

c) Incoming node

d) Outgoing node

6. Loop which do not possess any common node are said to be ___________ loops.

a) Forward gain

b) Touching loops

c) **Non touching loops**

d) Feedback gain

7. In force-current analogy, mass is analogous to

a) inductance

b) current

c) voltage

d) **capacitance**

8. In force-voltage analogy, moment of inertia is analogous to

a) capacitance
b) inverse capacitance
c) inductance
d) inverse inductance

9. Force balancing equation of a mass element is (where, x = displacement)

a) $M\ d^2x/dt^2$
b) $M\ dx/dt$
c) $M\ *x$
d) Any of the above

10. Force balancing equation for elastic element (K) is (where, x = displacement)

a) $K\ d^2x/dt^2$
b) $K\ dx/dt$
c) $K\ *x$
d) Any of the Above

Conclusion:

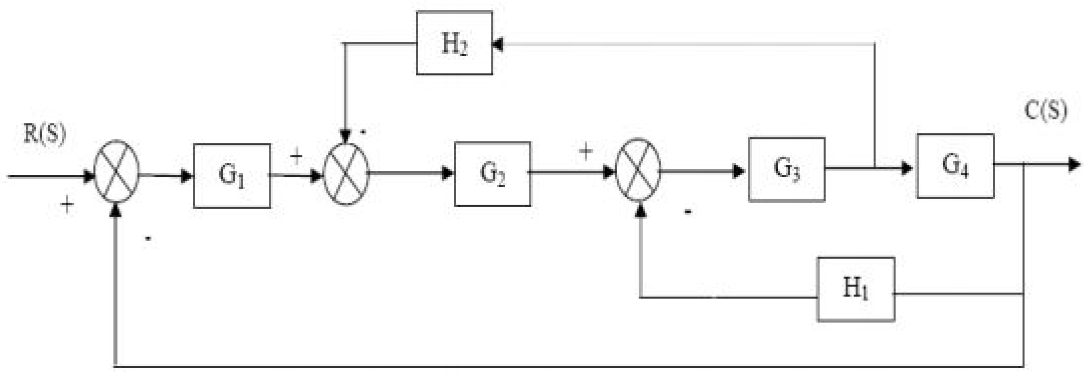
- Mathematical Modeling of Mechanical and Electrical System were studied
- Block Diagram Representation and Reduction were studied
- Signal Flow graph method were analyzed and compared with Block diagram reduction Technique

References:

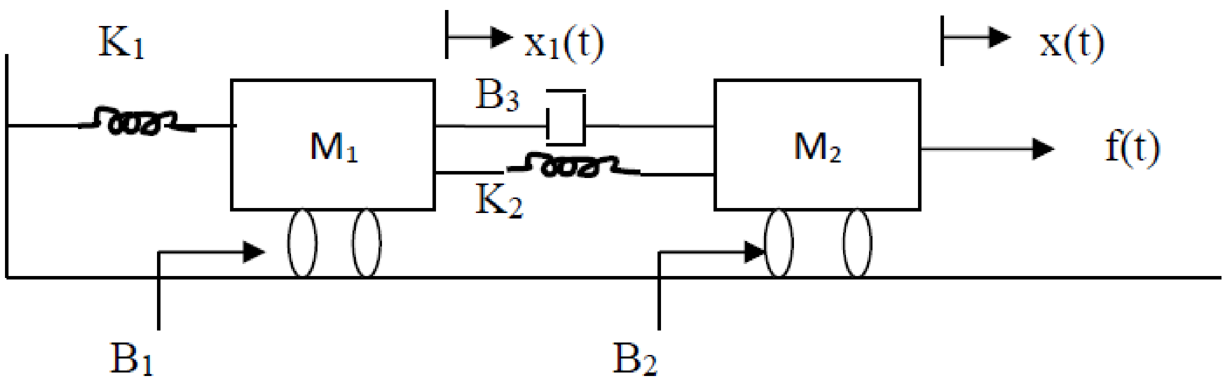
- M.Gopal,"Control system – Principle and Design," Tata McGraw Hill, 6th edition, 2012.
- K.Ogata,"Modern control Engineering, "fifth edition, PHI, 2012

Assignments

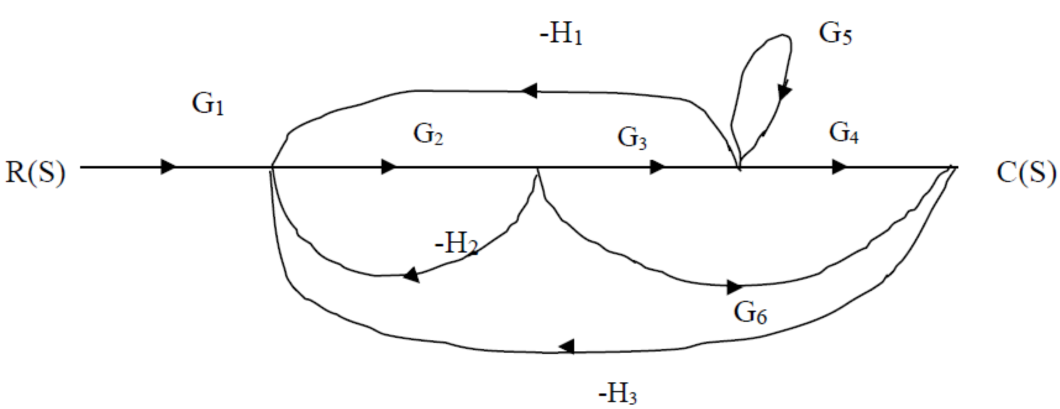
1. Find C(S)/R(S)using block diagram reduction technique



2. Obtain the transfer function F(s)/X(s) of the mechanical system



3. Use Manson’s gain formula for determining the overall transfer function of the system shown in the following figure



4. Draw the signal flow graph of the system represented by the block diagram & find the overall transfer function using Manson's gain formula